Financial Networks and Contagion

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- We study cascades of failures in a network of interdependent financial organizations: how discontinuous changes in asset values trigger further failures, and how this depends on network structure
- Integration (greater dependence on counterparties) and diversification (more counterparties per organization) have different, nonmonotonic effects on the extent of cascades
- Diversification connects the network initially, permitting cascades to travel; but as it increases further, organizations are better insured against one another's failures
- Integration also faces trade-offs: increased dependence on other organizations versus less sensitivity to own investment
- Finally, we illustrate the model with data on European debt cross-holdings

Introduction

- Globalization brings with it increased financial interdependencies among many kinds of organizations
- Such interdependencies can lead to cascading defaults and failures, which are often avoided through massive bailouts of institutions deemed "too big to fail"
- The US government's interventions in AIG, Fannie Mae, Freddie Mac, and General Motors; and the European Commission's interventions in Greece and Spain
- They emphasize the need to study the risks created by a network of interdependencies
- Understanding these risks is crucial to designing incentives and regulatory responses which defuse cascades before they are imminent

- We develop a general model that produces new insights regarding financial contagions and cascades of failures among organizations linked through a network of financial interdependencies
- Cross-holdings: Organizations' values depend on each other - e.g., through cross-holdings of shares, debt, or other liabilities
- Discontinuous losses: If an organization's value becomes sufficiently low, it discontinuously loses further value; these losses can propagate to others
- Hierarchies of cascades: Shocks are amplified in different stages

- Organizations hold primitive assets as well as shares in each other
- We derive the non-inflated "market value" that any organization delivers to final investors outside the system of cross-holdings
- How each organization's market value depends on the values of the primitive assets and on any failure costs that have hit the economy
- How asset values and failures costs propagate through the network of interdependencies
- Distinguish the sequence of dependencies in order to figure out how they might be avoided

- Introduces a variation of standard algorithm to compute
- Policymakers can use this algorithm to run counterfactual scenarios
- How the probability and extent of cascades depend on cross-holdings: integration and diversification
- Integration refers to the level of exposure of organizations to each other
- Diversification refers to how spread out cross-holdings are

Trade-offs of Integration and Diversification

- Although integration can increase the likelihood of a cascade once an initial failure occurs, it can also decrease the likelihood of that first failure
- Low diversification: organizations can be very sensitive to particular others, but the network of interdependencies is disconnected and overall cascades are limited
- A "sweet spot" of the level of diversification
- High diversification: organizations become insensitive to any particular organization's failure
- Two conditions for widespread financial cascades:
  1. Integration is intermediate
  2. Organizations are partly diversified
- Provides analytical results on a class of tractable network and simulation results on other random cross-holding networks

Policy Relevance

- What a regulator or government might do to mitigate the possibility of cascades of failures
- Preventing a first failure: a reallocation of cross-holdings does not work
- Bailing out the organization most at risk of failing is necessary
- In the end, we illustrate the model in the context of cross-holdings of European debt

Differences from the Literature

- Our methodology and results are different from the existing literature, especially the nonmonotonicities in cascades due to integration and diversification
- We distinguish integration and diversification
- We consider a class of random networks and ask how the consequences of a given moderate shock depend on diversification and integration
- The results highlight that intermediate levels of diversification and integration can be the most problematic

1. The Model and Determining Organizations' Values with Cross-Holdings

A. Primitive Assets, Organizations, and Cross-holdings

- There are n organizations making up a set $N = \{1, \dots, n\}$
- The values of organizations are ultimately based on the values of primitive assets or factors of production, $M = \{1, \dots, , \}$
- The market price of asset k is p_k
- $D_{ik} \geq 0$ is the share of the value of asset k held by organization i, and $\mathbf{D}$ is the matrix whose (i, k)th entry is equal to D_{ik}
- For any $i, j \in N$ the number $C_{ij} \geq 0$ is the fraction of organization j owned by organization i, where $C_{ii} = 0$ for each i
- The matrix $\mathbf{C}$ can be thought as a network in which there is a directed link from i to j if $C_{ij} > 0$. Ownership paths and cascade paths
- $\hat{C}_{ii} := 1 - \sum_{j \in N} C_{ji} > 0$ is the share of organization i not owned by any organization in the system

- Linear dependencies allow a tractable analysis of cross-dependencies, and provides basic insights and should still be useful when nonlinearities are addressed in detail

B. Values of Organizations: Accounting and Adjusting for Cross-Holdings

- The equity or book value V_i of an organization i is the total value of its shares
- This is equal to the value of organization i's primitive assets plus the value of its claims on other organizations:

$$V_i = \sum_k D_{ik} p_k + \sum_j C_{ij} V_j$$

- The matrix notation

$$\mathbf{V} = \mathbf{D}\mathbf{p} + \mathbf{C}\mathbf{V},$$

$$\mathbf{V} = (\mathbf{I} - \mathbf{C})^{-1} \mathbf{D}\mathbf{p}$$

- The sum of the V_i exceeds the total value of primitive assets held by the organizations

- The inflated value: each dollar of net primitive assets directly held by organization i contributes one dollar to the equity value of organization i, but is also counted partially on the books of all the organizations that have an equity stake in i
- The literature points out that the ultimate non-inflated value of an organization to the economy is well-captured by the equity value of that organization that is held by its outside investors
- This value captures the flow of real assets which accrues to final investors of that organization
- The market value $v_i = \hat{C}_{ii} V_i$, and therefore:

  $$\mathbf{v} = \hat{\mathbf{C}}\mathbf{V} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}\mathbf{D}\mathbf{p} = \mathbf{A}\mathbf{D}\mathbf{p}.$$

  where $\mathbf{A} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}$ as the dependency matrix
- This is reminiscent of Leontief's (1951) input-output analysis

C. Discontinuities in Values and Failure Costs

- Organizations can lose productive value in discontinuous ways if their values fall below certain critical thresholds
- These discontinuities can lead to cascading failures and also the presence of multiple equilibria
- Many sources of discontinuity
- If the value v_i of an organization i falls below some threshold level $\underline{v}_i$, then i is said to fail and incurs failure costs $\beta_i(\mathbf{p})$

D. Including Failure Costs in Market Values

- The book value of organization i considering the discontinuous drop becomes:

$$V_i = \sum_{j \neq i} C_{ij} V_j + \sum_k D_{ik} p_k - \beta_i I_{v_i < \underline{v}_i},$$

$$\mathbf{V} = (\mathbf{I} - \mathbf{C})^{-1}(\mathbf{Dp} - \mathbf{b}(\mathbf{v}, \mathbf{p})),$$

where $b_i(\mathbf{v}, \mathbf{p}) = \beta_i(\mathbf{p}) I_{v_i < \underline{v}_i}$.

$$\mathbf{v} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}(\mathbf{Dp} - \mathbf{b}(\mathbf{v})) = \mathbf{A}(\mathbf{Dp} - \mathbf{b}(\mathbf{v}, \mathbf{p}))$$

- A_{ij} describes the proportion of j's failure costs that i bears when j fails as well as i's claims on the primitive assets that j directly holds
- If j fails, thereby incurring failure costs of β_j, then i's value will decrease by $A_{ij}\beta_j$

E. A Simple Microfoundation

- The concrete process of liquidation and rationing of value
- $m = n$ and $\mathbf{D} = \mathbf{I}$
- If i liquidates its proprietary asset, it incurs a loss of λ_i cents on the dollar, $\beta_i(\mathbf{p}) = \lambda_i p_i$
- It follows that $\mathbf{v} = \mathbf{A}(\mathbf{p} - \mathbf{b}(\mathbf{v}, \mathbf{p}))$

F. Equilibrium Existence and Multiplicity

- A solution for organization values in $\mathbf{v} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}(\mathbf{Dp} - \mathbf{b}(\mathbf{v})) = \mathbf{A}(\mathbf{Dp} - \mathbf{b}(\mathbf{v}, \mathbf{p}))$ is an equilibrium set of values, and encapsulates the network of cross-holdings in a clean and powerful form, building on the dependency matrix $\mathbf{A}$
- There always exists a solution, and there can be multiple solutions. In fact, the set of solutions forms a complete lattice following Tarski's fixed point theorem
- Two sources of multiple equilibria: individual self-fulfilling bank runs and the interdependence of the values of the organizations
- We focus on the best-case equilibrium, in which as few organizations as possible fail
- This allows us to isolate sources of necessary cascades, distinct from self-fulfilling sorts of failure, which have already been studied in the sunspot and bank-run literatures

G. Measuring Dependencies

- The dependency matrix **A** takes into account all indirect holdings as well as direct holdings
- The central insights of the paper are derived using this matrix
- We identify some useful properties of the dependency matrix **A** and explore its relation to direct cross-holdings **C**

$$\mathbf{C} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$$

$$\widehat{\mathbf{C}} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$\mathbf{A} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

- LEMMA 1: $\hat{C}_{ii}$ is a lower bound on A_{ii} and A_{ii} can be much larger than $\hat{C}_{ii}$

H. Avoiding a First Failure

- There are trade-offs in preventing the spark that ignites a cascade
- Any fair trades of cross-holdings and assets that help an organization avoid failure in some circumstances must make it vulnerable to failure in some new circumstances
- "no-free-lunch" for avoiding first failures
- Fair trades are exchanges of cross-holdings or underlying assets which leave the market values of the organizations unchanged at current prices

PROPOSITION 1: *Suppose an organization i is closest to failing at asset prices* $\mathbf{p} > \mathbf{0}$, *cross-holdings* $\mathbf{C}$, *and direct holdings* $\mathbf{D}$. *Consider new cross-holdings and direct holdings* $\mathbf{C}'$ *and* $\mathbf{D}'$ *resulting from a fair trade at* $\mathbf{p}$ *such that row i of* $\mathbf{A}'$ *is different from that of* $\mathbf{A}$. *Then, for any* $\varepsilon > 0$, *there is a* $\mathbf{p}'$ *within an* ε*-neighborhood of* $\mathbf{q}(\mathbf{p}, \mathbf{C}, \mathbf{D}) > 0$,[33] *such that i fails at prices* $\mathbf{p}'$ *after the fair trade but not before:* $v_i(\mathbf{p}', \mathbf{C}', \mathbf{D}') < \underline{v}_i < v_i(\mathbf{p}', \mathbf{C}, \mathbf{D})$.

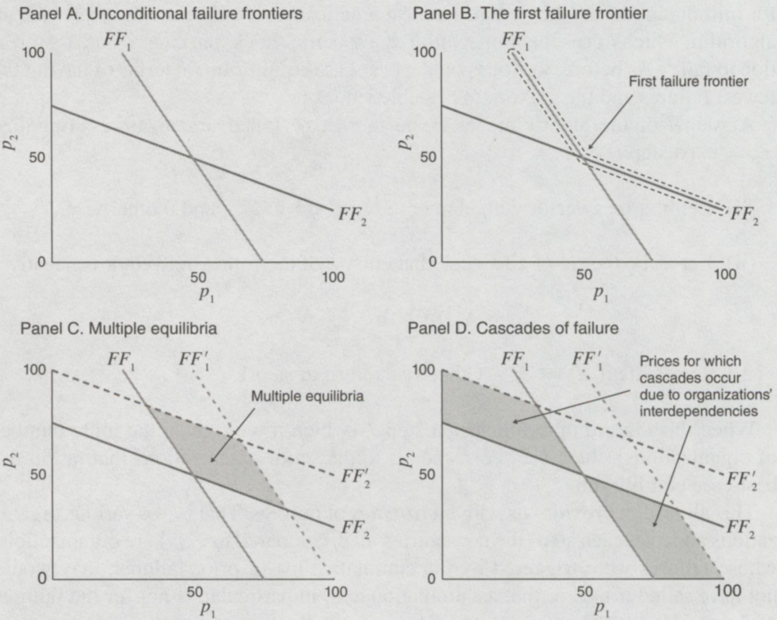
II. Cascades of Failures: Definitions and Preliminaries

A. Amplification through Cascades of Failures

- A relatively small shock to even a small organization can have large effects by triggering a cascade of failures
- Organization 1 has complete ownership of a single asset with value p_1
- $\mathbf{p}'$ differs from $\mathbf{p}$ only in the price of asset 1 such that $p_1' < p_1$
- Suppose $v_1(\mathbf{p}) > \underline{v}_1 > v_1(\mathbf{p}')$, so that 1 fails after the shock changing asset values from $\mathbf{p}$ to $\mathbf{p}'$
- 2's value also decreases by a term arising from 1's failure cost, $A_{21}\beta_1$
- If 2 also fails, 3 absorbs part of both failure costs: $A_{31}\beta_1 + A_{32}\beta_2$
- The cumulative failure costs to the economy of the first K organizations are $\beta_1 + \cdots + \beta_K$, which can greatly exceed the drop in asset value that precipitated the cascade

B. Who Fails in a Cascade?

- We study the best-case equilibrium following a shock to identify the minimal possible set of organizations that could fail
- Identifying who fails when using the dependency matrix **A**
- Suppose i fails when its value falls below 50 and upon failing incurs failure costs of 50. i therefore fails when $\frac{2}{3}p_i + \frac{1}{3}p_j < 50$
- If j fails, then i's value falls discontinuously since i bears one-third of j's failure costs of 50
- i fails if $\frac{2}{3}p_i + \frac{1}{3}(p_j - 50) < 50$. This new failure threshold is i's failure frontier conditional on j failing
- Multiple equilibria: both i and j survive or both i and j fail



Panel A. Unconditional failure frontiers
FF_1
FF_2
p_2
p_1
100
50
0
50
100
Panel B. The first failure frontier
FF_1
FF_2
First failure frontier
p_2
p_1
100
50
0
50
100
Panel C. Multiple equilibria
FF_1
FF'_1
FF'_2
FF_2
Multiple equilibria
p_2
p_1
100
50
0
50
100
Panel D. Cascades of failure
FF_1
FF'_1
FF'_2
FF_2
Prices for which cascades occur due to organizations' interdependencies
p_2
p_1
100
50
0
50
100

- A Simple Algorithm for Identifying Cascade Hierarchies:

  At step t of the algorithm, let $\mathcal{Z}_t$ be the set of failed organizations. Initialize $\mathcal{Z}_0 = \emptyset$. At step $t \geq 1$:

  (i) Let $\tilde{\mathbf{b}}_{t-1}$ be a vector with element $\tilde{b}_i = \beta_i$ if $i \in \mathcal{Z}_{t-1}$ and 0 otherwise.

  (ii) Let $\mathcal{Z}_t$ be the set of all k such that entry k of the following vector is negative:

  $$\mathbf{A}\left[\mathbf{Dp} - \tilde{\mathbf{b}}_{t-1}\right] - \underline{\mathbf{v}}.$$

  (iii) Terminate if $\mathcal{Z}_t = \mathcal{Z}_{t-1}$. Otherwise return to step 1.

- When this algorithm terminates at step T, the set $\mathcal{Z}_T$ corresponds to the set of organizations that fail in the best-case equilibrium
- The sets depend on **p**, **C**, and **D**, and so each configuration of these can result in a different structure of failures
- The hierarchical structure of failures has immediate and strong policy implications

C. Defining Integration and Diversification

- This paper contributes to the literature by distinguishing the roles of diversification and integration in cascades
- Formally, cross-holdings $\mathbf{C}'$ are more diversified than cross-holdings $\mathbf{C}$ if and only if
  1. $C'_{ij} \leq C_{ij}$ for all i, j such that $C_{ij} > 0$, with strict inequality for some ordered pair (i, j), and
  2. $C'_{ij} > C_{ij} = 0$ for some i, j
- Thus, diversification captures the spread in organizations' cross-holdings

- A financial system becomes more integrated if the external shareholders of each organization i have lower holdings, so that the total cross-holdings of each organization by other organizations weakly increase
- Formally, cross-holdings $\mathbf{C}'$ are more integrated than cross-holdings $\mathbf{C}$ if and only if $\hat{C}'_{ii} \leq \hat{C}_{ii}$ for all i, with strict inequality for some i
- $\sum_{j:j\neq i} C'_{ji} \geq \sum_{j:j\neq i} C_{ji}$ for all i, with strict inequality for some i
- Integration captures the depth or extent of organizations' cross-holdings
- Integration is an intensive margin, while diversification is an extensive margin

D. Essential Ingredients of a Cascade

- Three ingredients that are necessary for a widespread cascade:
  1. A First Failure: Some organization must be susceptible enough to shocks in some assets that it fails
  2. Contagion: It must be that some other organizations are sufficiently sensitive to the first organization's failure that they also fail
  3. Interconnection: It must be that the network of cross-holdings is sufficiently connected so that the failures can continue to propagate and are not limited to some small component
- Keeping these different ingredients of cascades in mind will help us disentangle the different effects of changes in cross-holdings

III. How do Cascades Depend on the Diversification and Integration of Cross-Holdings?

A. The Consequences of Diversification and Integration: Analytic Results

- Proposition 2: Consider (C, D) and (C', and D') that are related by a fair trade at **p**, and such that integration increases: $A'_{ij} \geq A_{ij}$ whenever $i \neq j$. There is then the same set of first failures at $(\mathbf{p}, \mathbf{C}, \mathbf{D})$ as at $(\mathbf{p}, \mathbf{C}', \mathbf{D}')$, and every organization that fails in a cascade at $(\mathbf{p}, \mathbf{C}, \mathbf{D})$ also fails at $(\mathbf{p}, \mathbf{C}', \mathbf{D}')$
- If we integrate cross-holdings via fair trades, so that organizations end up holding more of each other's investments, then we face more failures in any given cascade that begins
- Benefits of integration come only via avoiding first failures
- The trade-off: integrating can eliminate some first failures. However, given that a first failure occurs, integration only exacerbates the resulting cascade

- Proposition 3: If one proprietary asset fails (uniformly at random), a nonvanishing fraction of organizations fail if and only if there are intermediate levels of both integration and diversification
- It documents a non-monotonicity of failures in diversification and integration
- Intuition: If a fraction c, each firm is held by other organizations, is very low, then no firm holds enough of its counterparties for contagion to propagate
- If c is very high, then no firm is sufficiently exposed to its own asset for a first failure to happen
- If the average directed degree d is less than 1, contagion to a positive fraction of organizations is impossible
- If d is too large, a single organization organization's failure will not induce a second failure

- Simulated Random Networks. $m = n$, and $\mathbf{D} = \mathbf{I}$
- We start with asset values of $p_i = 1$ for all organizations, and have common failure thresholds $\underline{v}_i = \theta v_i$, for a parameter $\theta \in (0, 1)$, where v_i is the starting value of organization i when all assets are at value 1. An organization loses it full value when it fails, so that $\beta_i = \underline{v}_i$

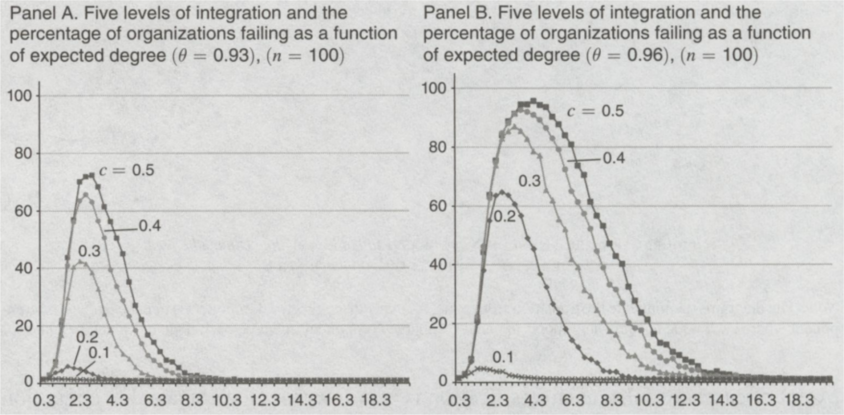


FIGURE 4. HOW INTEGRATION (THE FRACTION c OF A TYPICAL PORTFOLIO HELD BY OTHER ORGANIZATIONS) AFFECTS THE PERCENTAGE OF ORGANIZATIONS FAILING (*Averaged over 1,000 simulations*)

Notes: The x-axis corresponds to the diversification level (the expected degree in the random network of cross-holdings). The two figures work with different failure thresholds and depict how the size of cascades varies with the level of integration c ranging from 0.1 to 0.5.

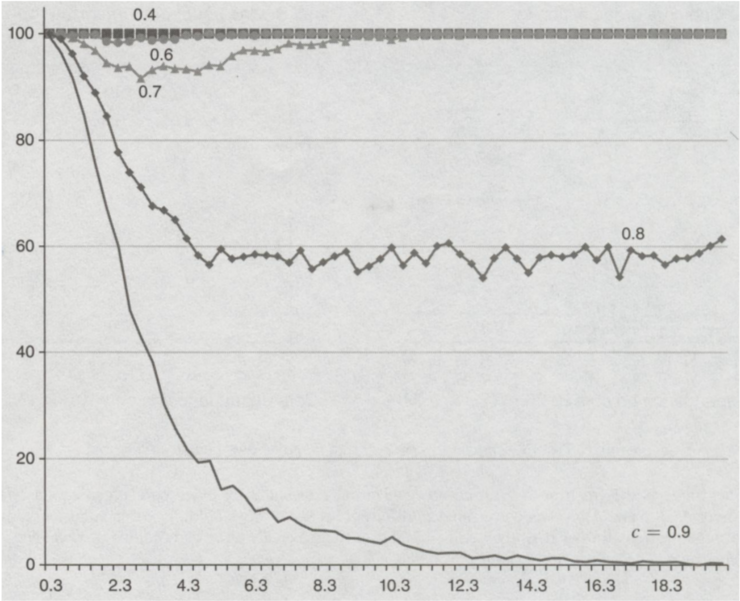


Figure 5. How Integration Affects the Percentage of "First Failures"

Notes: The percentage of simulations with at least one organization failing, for various levels of integration c from 0.4 to 0.9, with the x-axis tracking diversification (expected degree) in the network. The failure threshold is con-

IV. Alternative Network Structures

A. A Core-Periphery Model

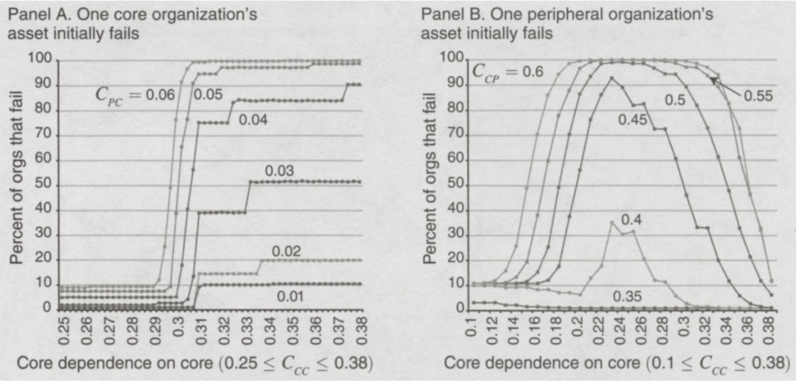


FIGURE 6. THE CONSEQUENCES OF FAILURE IN THE CORE-PERIPHERY MODEL

Notes: The *x*-axis is the fraction of each core organization cross-held by other core organizations (integration of core to core). In panel A, curves correspond to different levels of cross-holdings of each core organization by peripheral organizations. In panel B, they correspond to different levels of cross-holdings of peripheral organizations by core ones. The failure threshold is $\theta = 0.98$.

IV. Alternative Network Structures

B. A Model with Segregation among Sectors

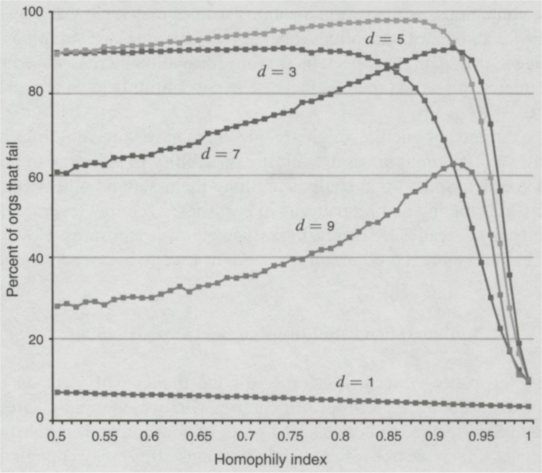


FIGURE 7. TEN GROUPS OF TEN ORGANIZATIONS EACH

Notes: The y-axis is the fraction of organizations that fail as a function of the homophily. The x-axis is the fraction of expected cross-holdings in same-type organizations. Curves correspond to different diversification levels (expected degrees d). The failure threshold is $\theta = 0.96$.

- Power Law Distributions (more extreme degree distributions): More extreme exponents in the power law actually lead to smaller contagions on average, but also lead to larger contagions conditional on some high-degree organization's failure
- Correlated and Common Assets: Increasing correlation increases the failure rate. The more interesting part is that the increase occurs abruptly at a particular level of correlation

V. Illustration with European Debt Cross-Holdings

	(France)	(Germany)	(Greece)	(Italy)	(Portugal)	(Spain)
(France)	0	198,304	39,458	329,550	21,817	115,162
(Germany)	174,862	0	32,977	133,954	30,208	146,096
(Greece)	1,960	2,663	0	444	51	292
(Italy)	40,311	227,813	2,302	0	3,188	26,939
(Portugal)	6,679	2,271	8,077	2,108	0	21,620
(Spain)	27,015	54,178	1,001	29,938	78,005	0

To convert the above matrix into our fractional cross-holdings matrix, **C**, we then estimate the total amount of debt issued by each country. To do this, we estimate the ratio of total debt held outside the issuing country by 1/3, in line with estimates by Reinhart and Rogoff (2011). Then, the formula $\mathbf{A} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}$ implies that **A** is:

	(France)	(Germany)	(Greece)	(Italy)	(Portugal)	(Spain)
(France)	0.71	0.13	0.13	0.17	0.07	0.11
(Germany)	0.18	0.72	0.12	0.11	0.09	0.14
(Greece)	0.00	0.00	0.67	0.00	0.00	0.00
(Italy)	0.07	0.12	0.03	0.70	0.03	0.05
(Portugal)	0.01	0.00	0.02	0.00	0.67	0.02
(Spain)	0.03	0.03	0.02	0.02	0.14	0.68

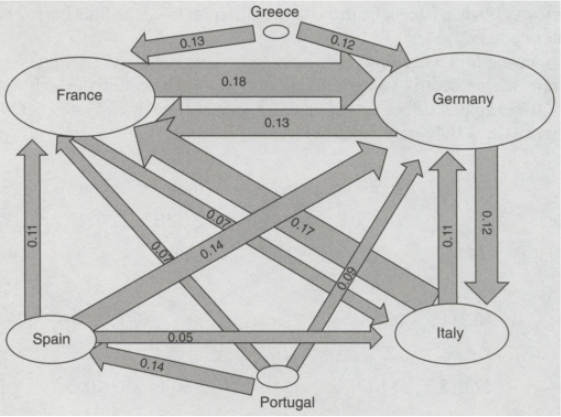


FIGURE 8. INTERDEPENDENCIES IN EUROPE

Notes: The matrix **A**, describing how much each country ultimately depends on the value of others' debt. The widths of the arrows are proportional to the sizes of the dependencies, with dependencies less than 5 percent excluded; the area of the oval for each country is proportional to its underlying asset values.

We treat the investments in primitive assets as if each country holds its own fiscal stream, which is used to pay for the debt, and presume that the values of these fiscal streams are proportional to GDP (gross domestic product). Thus, $\mathbf{D} = \mathbf{I}$ and $\mathbf{p}$ is proportional to the vector of countries' GDPs.[61] Normalizing Portugal's 2011 GDP to 1, the initial values in 2011 are $\mathbf{v}_0 = \mathbf{A}\mathbf{p}$,

$$\begin{pmatrix} 0.71 & 0.13 & 0.13 & 0.17 & 0.07 & 0.11 \\ 0.18 & 0.72 & 0.12 & 0.11 & 0.09 & 0.14 \\ 0.00 & 0.00 & 0.67 & 0.00 & 0.00 & 0.00 \\ 0.07 & 0.12 & 0.03 & 0.70 & 0.03 & 0.05 \\ 0.01 & 0.00 & 0.02 & 0.00 & 0.67 & 0.02 \\ 0.03 & 0.03 & 0.02 & 0.02 & 0.14 & 0.68 \end{pmatrix} \cdot \begin{pmatrix} 11.6 \\ 14.9 \\ 1.3 \\ 9.2 \\ 1.0 \\ 6.3 \end{pmatrix} = \begin{pmatrix} 12.7 \text{ (France)} \\ 14.9 \text{ (Germany)} \\ 0.8 \text{ (Greece)} \\ 9.4 \text{ (Italy)} \\ 0.9 \text{ (Portugal)} \\ 5.4 \text{ (Spain)} \end{pmatrix}.$$

Table 1—Hierarchies of Cascades in the Best-Case Equilibrium Algorithm, as a Function of the Failure Threshold θ

Value of θ	0.9	0.93	0.935	0.94
First failure	Greece	Greece	Greece	Greece
Second failure			Portugal	Portugal, Spain
Third failure			Spain	France, Germany
Fourth failure			France	Italy
Fifth failure			Germany, Italy	

Source: Authors' calculations

VI. Concluding Remarks

- Based on a simple model of cross-holdings among organizations that allows dis-continuities in values, we have examined cascades in financial networks
- First, diversification and integration are usefully distinguished as they have different effects on financial contagions
- Second, both diversification and integration entail trade-offs in how they affect contagion, resulting in non-monotonic effects where middle ranges are the most dangerous

Comments

- Wordiness and repetitiveness in writing
- This paper supports governments' bailing out of financial institutions, while the moral hazard cost of financial institution is concerning but is not included in the model
- I learn from this paper about how to highlight its differences from existing literature
- I am working on modeling when and how decentralized finance can mitigate the cascades of financial failures